CHAPTER 2

# MATRICES

## POINTS TO REMEMBER

Matrix : A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements of the matrix.

Order of Matrix : A matrix having 'm' rows and 'n' columns is called the matrix of order mxn.

Zero Matrix : A matrix having all the elements zero is called zero matrix or null matrix.

Diagonal Matrix : A square matrix is called a diagonal matrix if all its non diagonal elements are zero.

Scalar Matrix : A diagonal matrix in which all diagonal elements are equal is called a scalar matrix.



Identity Matrix : A scalar matrix in which each diagonal element is 1, is called an identity matrix or a unit matrix. It is denoted by I.

$\therefore \quad I = [e_{ij}]_{n \times n}$

where, $e_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

Transpose of a Matrix : If $A = [a_{ij}]_{m \times n}$ be an $m \times n$ matrix then the matrix obtained by interchanging the rows and columns of A is called the transpose of the matrix. Transpose of A is denoted by $A'$ or $A^T$.

Properties of the transpose of a matrix.

(i) $(A')' = A$ (ii) $(A + B)' = A' + B'$

(iii) $(KA)' = KA'$, K is a scalar (iv) $(AB)' = B'A'$

Symmetrix Matrix : A square matrix $A = [a_{ij}]$ is symmetrix if $a_{ij} = a_{ji}$ $\forall$ i, j. Also a square matrix A is symmetrix if $A' = A$.

Skew Symmetrix Matrix : A square matrix $A = [a_{ij}]$ is skew-symmetrix, if $a_{ij} = -a_{ji}$ $\forall$ i, j. Also a square matrix A is skew - symmetrix, if $A' = -A$.

Determinant : To every square matrix $A = [a_{ij}]$ of order $n \times n$, we can associate a number (real or complex) called determinant of A. It is denoted by det A or $|A|$.

Properties

(i) $|AB| = |A|\,|B|$

(ii) $|KA|_{n \times n} = K^n\,|A|_{n \times n}$ where K is a scalar.

Area of triangles with vertices $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

The points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are collinear $\Leftrightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Adjoint of a square matrix A is the transpose of the matrix whose elements have been replaced by their cofactors and is denoted as adj A.

Let $A = [a_{ij}]_{n \times n}$

$\text{adj } A = [A_{ji}]_{n \times n}$ where $A_{ji}$ are co-factor of element $a_{ji}$

Properties

(i) $A(\text{adj } A) = (\text{adj } A)A = |A|\,I$

(ii) If A is a square matrix of order n then $|\text{adj } A| = |A|^{n-1}$

(iii) $\text{adj } (AB) = (\text{adj } B)(\text{adj } A)$.

Singular Matrix : A square matrix is called singular if $|A| = 0$, otherwise it will be called a non-singular matrix.

Inverse of a Matrix : A square matrix whose inverse exists, is called invertible matrix. Inverse of only a non-singular matrix exists. Inverse of a matrix A is denoted by $A^{-1}$ and is given by

$$A^{-1} = \frac{1}{|A|}\,\text{adj}\,A$$



Properties

(i) $AA^{-1} = A^{-1}A = I$

(ii) $(A^{-1})^{-1} = A$

(iii) $(AB)^{-1} = B^{-1}A^{-1}$

(iv) $(A')^{-1} = (A^{-1})'$

Solution of system of equations using matrix :

1) If $AX = B$ is a matrix equation then its solution is $X = A^{-1}B$.

(i) If $|A| \neq 0$, system is consistent and has a unique solution.

(ii) If |A| = 0 and (adj A) B ≠ 0 then system is inconsistent and has no solution.

(iii) If |A| = 0 and (adj A) B = 0 then system is consistent and has infinite solution.

2) If $AX = 0$ is a matrix equation,where is a squqre matrix

.(i) If $|A| \neq 0$,then system has trivial solution.

(ii) If $|A| = 0$ then system has infinitely many solutions.

Note- If $|A| = 0$, then $(adj\ A)\ B = 0$ as $B = 0$

VERY SHORTANSWER TYPE QUESTIONS (1Mark)

1. Let A be a square matrix of order $3 \times 3$ than $|KA|$ is equal to :

(A) $k|A|$ (B) $K^2|A|$ (C) $k^3|A|$ (D) $3k|A|$

2. If a, b, c are in A.P. then determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \text{ is}$$

(A) 0 (B) 1 (C) $x$ (D) $2x$

3. $T_p, T_q, T_r$ are the p$^{th}$, q$^{th}$ and r$^{th}$ terms of an A.P. then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

(A) 1 (B) -1 (C) 0 (D) p+q+r

4. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ , $\omega$ being a cube root of unity is

(A) 0 (B) 1 (C) $\omega^2$ (D) $\omega$

5. If a+b+c=0, one root of

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

(A) $x$=1 (B) $x$=2 (C) $x = a^2 + b^2 + c^2$ (D) $x$=0

6. The roots of the equation

$$\begin{vmatrix} a-x & b & c \\ 0 & b-x & 0 \\ 0 & b & c-x \end{vmatrix} = 0 \text{ are}$$

(A) a and b (B) b and c (C) a and c (D) a,b and c

7. Value of $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is

(A) (a-b) (b-c) (c-a) (B) $(a^2-b^2)(b^2-c^2)(c^2-a^2)$

(C) (a-b+c) (b-c+a) (c-a+b) (D) None of these

8. If A and B are any 2 x 2 matrices, then det (A+B)=0 implies

(A) det A + det B=0 (B) det A=0 or det B=0

(C) det A=0 and det B=0 (D) None of these

9. If A and B are 3 x 3 matrices then AB=0 implies

(A) A=0 and B=0 (B) |A|=0 and |B|=0

(C) Either |A|=0 or |B|=0 (D) A=0 or B=0

10. The value of $\lambda$ for which the system of equations :-

$x+y+2=6$, $x+2y+3z=10$, $x+4y+\lambda z=1$ has a unique solution is

(A) $\lambda \neq -7$ (B) $\lambda \neq 7$

(C) $\lambda = 7$ (D) $\lambda = -7$



11. If the system of the equation :

$x-ky-z=0$, $kx-y-z=0$, $x+y-z=0$ has a non-zero solution, then the possible values of $k$ are :

(A) -1, 2 (B) 1,2

(C) 0, 1 (D) -1, 1

12. If A is a 3 x 3 non singular matrix than det [adj. (A)] is equal to

(A) $(\det A)^2$ (B) $(\det A)^3$

(C) $\det A$ (D) $(\det A)^{-1}$

13. If A is an invertible matrix of order $n$, then the determinant of Adj. A =

(A) $|A|^n$ (B) $|A|^{n+1}$

(C) $|A|^{n-1}$ (D) $|A|^{n+2}$

14. The value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

(A) a+b+c (B) 1

(C) 0 (D) abc

15. If $A^2-A+I=0$ then the inverse of A is

(A) A (B) A+I

(C) I-A (D) A-I



## SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1. Construct a $3 \times 2$ matrix A = $[a_{ij}]$ whose elements are given by $a_{ij} = \begin{cases} \frac{i+j}{2} & \text{if } i = j \\ \frac{|i-2j|}{2} & \text{if } i < j \end{cases}$

2. Find A and B if $2A + 3B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 6 & 2 \end{bmatrix}$.

3. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$, verify that $(AB)' = B'A'$.

4. Express the matrix $\begin{bmatrix} 3 & 3 & 1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = P + Q$ where P is a symmetric and Q is a skew-symmetric matrix.

5 If A= $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, verify prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ where n is a natural number.

6. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix D such that CD – AB = O.

7. Find the value of x such that $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix} = 0$

8. Prove that the product of the matrices $\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and $\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$ is the null matrix, when $\theta$ and $\phi$ differ by an odd multiple of $\frac{\pi}{2}$.

9. If $A = \begin{bmatrix}5 & 3\\ 12 & 7\end{bmatrix}$ show that $A^2 - 12A - I = 0$. Hence find $A^{-1}$.

10. If $A = \begin{bmatrix}2 & 3\\ 4 & 7\end{bmatrix}$ find f(A) where $f(x) = x^2 - 5x - 2$.

11. If $A = \begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$, find x and y such that $A^2 - xA + yI = 0$.

12. Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$.

13. If $A = \begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$ and $B = \begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}$ then show that $(AB)^{-1} = B^{-1}A^{-1}$.

14. Test the consistency of the following system of equations by matrix method : 3x – y = 5; 6x – 2y = 3

15. Using elementary row transformations, find the inverse of the matrix $A = \begin{bmatrix}6 & -3\\ -2 & 1\end{bmatrix}$, if possible.

16. By using elementary column transformation, find the inverse of $A = \begin{bmatrix}3 & 1\\ 5 & 2\end{bmatrix}$.

17. If $A = \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ and $A + A' = I$, then find the general value of $\alpha$.

Using properties of determinants, prove the following : Q 18 to Q 24

18. $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$ if a, b, c are in A.P.

19. $\begin{vmatrix} \sin [redacted] & \cos [redacted] & \sin ([redacted]) \\ \sin [redacted] & \cos [redacted] & \sin ([redacted]) \\ \sin [redacted] & \cos [redacted] & \sin ([redacted]) \end{vmatrix} = 0$

20. $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab^2 & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$ ac [redacted] c [redacted] 2

20. $a^2 + ab$ $b^2$ ac $4a^2b^2c^2$.

ab b 2+ bc c 2

21. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(a+b+c+d)$

x + a b c

.21 a a b x+b x + c c $x^2$ ( [redacted] + a + b [redacted] )

a b x + c

22. Show that :

$$\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy).$$

23. (i) If the points (a, b) (a′, b′) and (a – a′, b – b′) are collinear. Show that ab′ = a′b.

(ii) If $A [redacted] \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B [redacted] \begin{bmatrix} 4 & [redacted]3 \\ 2 & 5 \end{bmatrix}$ verify that [redacted] AB [redacted] A [redacted] B [redacted].

24 Solve the following equation for x.

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

## LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. Obtain the inverse of the following matrix using elementary row operations $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

2. Using matrix method, solve the following system of linear equations :

$x - y + 2z = 1,\ 2y - 3z = 1,\ 3x - 2y + 4z = 2.$

3. Solve the following system of equations by matrix method, where $x \neq 0, y \neq 0, z \neq 0$

$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10,\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10,\ \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13.$ where $x \neq 0, y \neq 0, z \neq 0$

4. Find $A^{-1}$, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, hence solve the system of linear equations :

$x + 2y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 11$

5. The sum of three numbers is 2. If we subtract the second number from twice the first number, we get 3. By adding double the second number and the third number we get 0. Represent it algebraically and find the numbers using matrix method.

6. Compute the inverse of the matrix.

$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 5 \end{bmatrix}$ and verify that $A^{-1}A = I_3$.

7. If the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ -1 & 0 & 2 \end{bmatrix}$, then compute $(AB)^{-1}$.

8. Using matrix method, solve the following system of linear equations : $2x - y = 4$, $2y + z = 5$, $z + 2x = 7$

9 Find the inverse of the matrix $A = \begin{vmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \end{vmatrix}$ by using elementary column transformations..

10. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find $A^5$

11. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, verify that $A . (\text{adj } A) = (\text{adj } A) . A = |A| I_3$.

12. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$, hence find $A^{-1}$.

13. By using properties of determinants prove the following :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

14. $$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^2.$$

15. $$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$$

16. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$ is equal to zero. Show that $xyz = -1$.